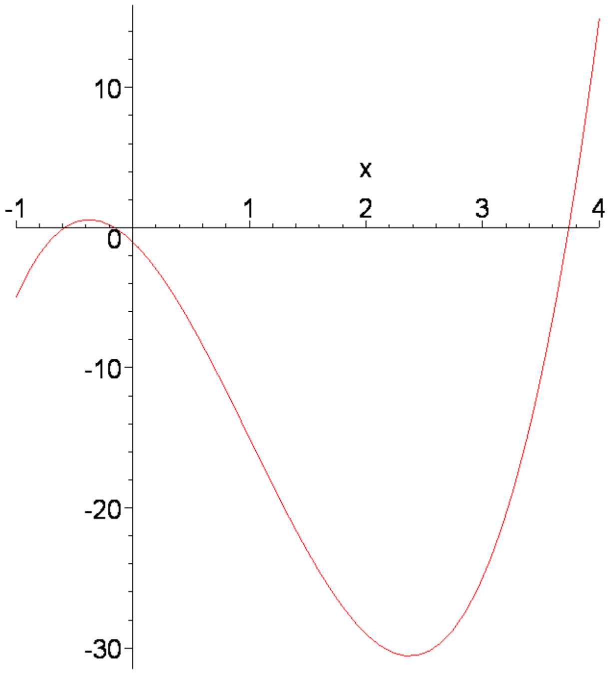
Vybrané metody aproximace reálných kořenů

ÚKOL: Proveďte aproximaci jednoho z kořenů daného polynomu f(x).

```
> restart;
> f:=3*x^3-9*x^2-8*x-1;
```

$$f := 3\,x^3 - 9\,x^2 - 8\,x - 1$$

```
> plot(f,x=-1..4);
```




```
> factor(f,complex);
```

$$3.\,(x + 0.5848876912)\,(x + 0.1524892949)\,(x - 3.737376986)$$

```
> f:=unapply(f,x);
```

$$f := x \rightarrow 3\,x^3 - 9\,x^2 - 8\,x - 1$$

```
> Tabulka:=matrix([['x','f(x)'],seq([x,f(x)],x=-4..4)]);
```

Tabulka :=

x	f(*x*)
-4	-305
-3	-139
-2	-45
-1	-5
0	-1
1	-15
2	-29
3	-25
4	15

Budeme aproximovat kladný kořen polynomu, který leží v intervalu (3,4):

I. Metoda půlení intervalu

```
> a:=3: b:=4:
> f(a)*f(b);
```

$$-375$$

```
> m:=(a+b)/2: evalf(m);
```

$$3.500000000$$

#

```
> f(a)*f(m);
```

$$\frac{2125}{8}$$

```
> a:=m: m:=(a+b)/2: evalf(m);
```

$$3.750000000$$

#

```
> f(a)*f(m);
```

$$\frac{-3485}{512}$$

```
> b:=m: m:=(a+b)/2: evalf(m);
```

$$3.625000000$$

#

```
> f(a)*f(m);
```

$$\frac{233325}{4096}$$

```
> a:=m: m:=(a+b)/2: evalf(m);
```

$$3.687500000$$

#

```
> f(a)*f(m);
```

$$\frac{27600975}{2097152}$$

```
> a:=m: m:=(a+b)/2: evalf(m);
```

$$3.718750000$$

#

```
> f(a)*f(m);
```

$$\frac{306747885}{134217728}$$

```
> a:=m: m:=(a+b)/2: evalf(m);
```

$$3.734375000$$

#

```
> f(a)*f(m);
```

$$\frac{1209144945}{8589934592}$$

```
> a:=m: m:=(a+b)/2: evalf(m);
```

$$3.742187500$$

II. Newtonova metoda

```
> x0:=4;
```

$$x0 := 4$$

```
> x1:=x0-f(x0)/D(f)(x0): evalf(x1);
```

3.765625000

```
> x2:=x1-f(x1)/D(f)(x1): evalf(x2);
```

3.737758826

```
> x3:=x2-f(x2)/D(f)(x2): evalf(x3);
```

3.737377057

```
> x4:=x3-f(x3)/D(f)(x3): evalf(x4);
```

3.737376986

```
> x5:=x4-f(x4)/D(f)(x4): evalf(x5);
```

3.737376986

Jiná volba počátečního bodu:

```
> x0:=3;
```

$$x0 := 3$$

```
> x1:=x0-f(x0)/D(f)(x0): evalf(x1);
```

4.315789474

```
> x2:=x1-f(x1)/D(f)(x1): evalf(x2);
```

3.852123125

```
> x3:=x2-f(x2)/D(f)(x2): evalf(x3);
```

3.743308951

```
> x4:=x3-f(x3)/D(f)(x3): evalf(x4);
```

3.737394099

```
> x5:=x4-f(x4)/D(f)(x4): evalf(x5);
```

3.737376986

```
> x6:=x5-f(x5)/D(f)(x5): evalf(x6);
```

3.737376986

IV. Metoda iterace

```
> f:=3*x^3-9*x^2-8*x-1;
```

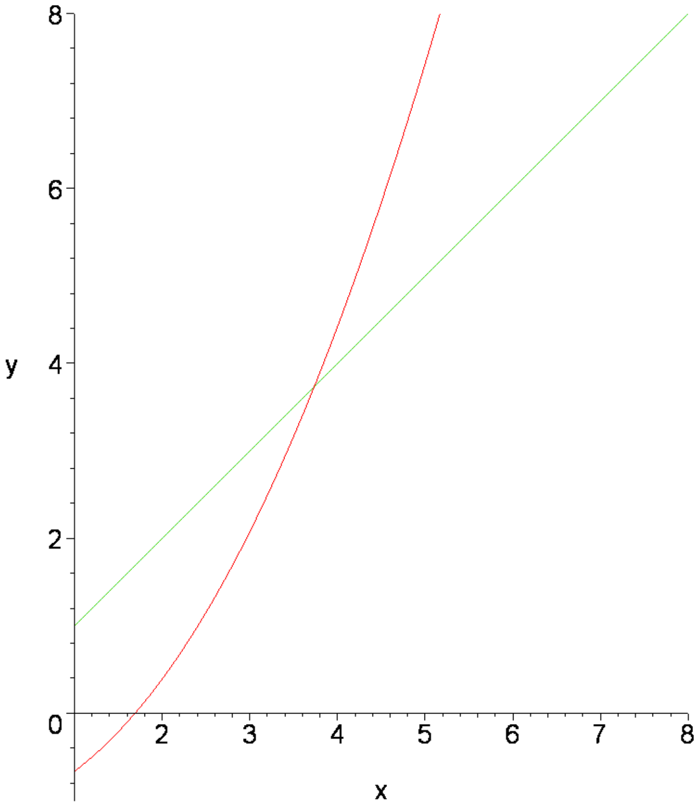
$$f := 3x^3 - 9x^2 - 8x - 1$$

```
> f_x:=x->x; f_g:=x->1/9*(3*x^2-8-1/x);
```

$$f\_x := x \rightarrow x$$

$$f\_g := x \rightarrow \frac{1}{3}x^2 - \frac{8}{9} - \frac{1}{9}\frac{1}{x}$$

```
> plot({f_x(x),f_g(x)},x=1..8,y=-1..8,scaling=constrained);
```




```
> x0:=4; f_g(x0); evalf(f_g(x0));
```

$$x0 := 4$$

$$\frac{53}{12}$$

$$4.416666667$$

```
> x1:=f_g(x0); f_g(x1); evalf(f_g(x1));
```

$$x1 := \frac{53}{12}$$

$$\frac{127949}{22896}$$

$$5.588268693$$

```
>
>
```